RTE
A COMPUTER CODE FOR THREE-DIMENSIONAL ROCKET THERMAL EVALUATION

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Regeneratively Cooled Rocket Engines
Modes of Heat Transfer Incorporated in RTE

- Convection and radiation from combustion gases (hot-gases).
- Three-dimensional conduction within the wall.
- Convection to the coolant.
- Conjugating all these modes of heat transfer.
Typical Nozzle Broken into a Number of Stations

Hot-gas in

Coolant out toward injector

Coolant in

Hot-gas out
Convection from Hot-Gases

- RTE uses the Chemical Equilibrium program by Gordon & McBride for hot-gas side thermodynamics and transport properties.
- Uses adiabatic wall temperature (enthalpy) along with convective heat transfer correlation for calculating heat flux from hot-gases.
- Can be linked to TDK’s Boundary Layer Modules for hot-gas-side heat flux calculations.
- Heat fluxes can be input using a matrix of heat fluxes.
Hot-Gas-Side Heat flux Using Adiabatic Wall Temperature (Enthalpy)

The reference enthalpy of the gas side, is given by (Eckert):

\[ i_{GX_n} = 0.5(i_{GW_n} + i_{GS_n}) + 0.180(i_{G0_n} - i_{GS_n}) \]

The adiabatic wall enthalpy (Bartz and Eckert)

\[ i_{GAW_n} = i_{GS_n} + (Pr_{GX_n})^{1/3}(i_{G0_n} - i_{GS_n}) \]
Hot-Gas Side Convective Heat Transfer

\[ q_n = h_{G_n} (T_{GAW_n} - T_{GW_n}) \]

Or

\[ q_n = \frac{h_{G_n}}{C_{p_{GX_n}}} (i_{GAW_n} - i_{GW_n}) \]
Hot-Gas Side Convective Heat Transfer

\[ h_{G_n} = \frac{C_{G_n} k_{GX_n}}{d_{G_n}} \text{Re}_{GX_n}^{0.8} \text{Pr}_{GX_n}^{0.3} \]

\[ \text{Re}_{GX_n} = \frac{4 \dot{W}_G}{\pi d_{G_n} \mu_{GX_n} T_{GS_n}} \frac{T_{GS_n}}{T_{GX_n}} \]

\[ \text{Pr}_{GX_n} = \frac{C_{p_{CX_n}} \mu_{GX_n}}{k_{GX_n}} \]
RTE-TDK Interface
for Boundary Layer Hot-Gas-Side Heat Flux Calculations

Start

- Run RTE with its internal heat flux model

- Run RTE-TDK interface program and print wall temperatures into TDK input

- Run TDK

- Run TDK-RTE interface program and print wall heat fluxes into RTE input

- Run RTE with known wall flux option

- Run RTE-TDK interface program and print wall temperatures into TDK input. Also, check for convergence

Convergence?

- Yes

- No

STOP
Inputting Hot-Gas-Side Heat Fluxes Using a Matrix of Fluxes

- Hot-gas-side heat fluxes can be input using a matrix of heat fluxes.
- Rows of this matrix represent axial positions and its columns represent wall temperatures.
- This option can be used if other programs are available for hot-gas-side heat flux calculations.
Radiation Heat Transfer from Hot-Gases

Radiation heat transfer is calculated using the Discrete Exchange Factor (DEF) method

\[ q''_r = \left( \sum_{l=1}^{m+2} w_{s_l} E_{s_l} \overline{DS_l S_n} + \sum_{l=1}^{m} w_{g_l} E_{g_l} \overline{DG_l S_n} - E_{s_n} \right) \]

\[ E_{s_n} = \varepsilon \sigma T_{s_n}^4 \quad E_{g_l} = 4K_{t_l} (1 - \omega_0) \sigma \pi r^2 T_{g_l}^4 \]
Cross-Section at Each Station

Cooling Channels
Coating

Close-up

NICKEL
COPPER
COATING
COOLING CHANNEL
INSULATION
3-D Finite Difference Method for Wall Heat Conduction

Each node at this station (station n) is linked to the two corresponding nodes at stations before (n-1) and after (n+1) for three dimensional heat conduction analysis.
3-D Finite Difference Model for Wall Heat Conduction

Typical middle node

\[ T_{i,j,n} = \frac{T_{i+1,j,n} / R_1 + T_{i,j-1,n} / R_2 + T_{i-1,j,n} / R_3 + T_{i,j+1,n} / R_4 + T_{i,j,n+1} / R_5 + T_{i,j,n-1} / R_6}{1 / R_1 + 1 / R_2 + 1 / R_3 + 1 / R_4 + 1 / R_5 + 1 / R_6} \]

\[ R_1 = \frac{r \Delta \phi}{\Delta r (\Delta S_{i,j}^{n-1,n} + \Delta S_{i,j}^{n,n+1})} \left( \frac{1}{k_{i,j,n}^{l-1}} + \frac{1}{k_{i+1,j,n}^{l-1}} \right) \]

\[ R_3 = \frac{r \Delta \phi}{\Delta r (\Delta S_{i,j}^{n-1,n} + \Delta S_{i,j}^{n,n+1})} \left( \frac{1}{k_{i,j,n}^{l-1}} + \frac{1}{k_{i-1,j,n}^{l-1}} \right) \]

\[ R_2 = \frac{\Delta r}{\left( r + \frac{\Delta r}{2} \right) \Delta \phi (\Delta S_{i,j}^{n-1,n} + \Delta S_{i,j}^{n,n+1})} \left( \frac{1}{k_{i,j,n}^{l-1}} + \frac{1}{k_{i,j,n-1}^{l-1}} \right) \]

\[ R_4 = \frac{\Delta r}{\left( r - \frac{\Delta r}{2} \right) \Delta \phi (\Delta S_{i,j}^{n-1,n} + \Delta S_{i,j}^{n,n+1})} \left( \frac{1}{k_{i,j,n}^{l-1}} + \frac{1}{k_{i,j+1,n}^{l-1}} \right) \]
3-D Finite Difference Method for Wall Heat Conduction

Typical surface node

\[ T_{i,j,n}^l = [T_{i+1,j,n}^{l-1} / R_1 + T_{i,j-1,n}^{l-1} / R_2 + T_{i-1,j,n}^{l-1} / R_3 + T_{i,j+1,n}^{l-1} / R_4 + T_{i,j,n+1}^{l-1} / R_5 + T_{i,j,n-1}^{l-1} / R_6 + Q_c + Q_r ]/(1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + 1/R_5 + 1/R_6) \]

\[ Q_r = \frac{\Delta \phi (\Delta S_{i,j}^{n-1,n} + \Delta S_{i,j}^{n,n+1}) \sin \beta_n}{4\pi} \left( \sum_{l=1}^{m+2} w_{s_l} E_{s_l} D S_l S_n + \sum_{l=1}^{m} w_{g_l} E_{g_l} D G_l S_n - E_{s_n} \right) \]
Wall Materials Incorporated in RTE

- Copper
- Nickel
- Soot
- NARloy-Z
- Columbium
- Zirconia

- SS-347
- Amzirc
- Platinum
- Glidcop
- Inconel718
- Nicraly
Coolant Flow Convection

- GASP (Gas Properties) and WASP (Water and Steam Properties) are used for evaluating coolant properties.
- A one-dimensional approach is used for coolant heat transfer and pressure drop calculations.
Convection Options

- Surface roughness
- Entrance effect
- Curvature effect
- Swiler option for enhancing coolant convection
- Cooling channel contraction and expansion
- Pass-and-a-half cooling channel option
- Pressure drop
- Blocked channel option to study a worst case scenario
Cooling Channel Heat Transfer Coefficient

\[
\frac{Nu}{Nu_r} = C_{C_n} Re^{0.8} Pr^{0.4}
\]

Where

\[
Nu_r = \psi^{-0.55} \quad \psi = 1 + \gamma (T_W - T_S)
\]

\[
\gamma = \left| \frac{1}{\rho} \frac{\partial \rho}{\partial T} \right|_P = \frac{1}{\rho} \left( \frac{\partial P}{\partial T} \right)_\rho
\]

\[
= \frac{1}{\rho} \left( \frac{\partial P}{\partial \rho} \right)_T
\]
Heat Transfer Coefficient for Oxygen

\[ Nu = C_{Cn} \frac{Re_{CS} Pr_{CS}^{0.4}}{\left( \frac{\bar{c}}{c_{PCS}} \right)^{0.2} \sqrt{\left( \frac{k_{CS}}{k_{CW}} \right) \left( \frac{\rho_{CW}}{\rho_{CS}} \right)}} \]

Where \( P_{Cri} = 731.4 \) psia

\[ \bar{c}_p = \frac{i_{CW} - i_{CS}}{T_{CW} - T_{CS}} \]
Entrance Effect Correlation
Options

\[
\phi_{\text{Ent.}} = 2.88 \left\{ \frac{\sum_{i=1}^{n} \Delta S_{i,i+1}}{d_{c_n}} \right\}^{-0.325}
\]

\[
\phi_{\text{Ent.}} = 1 + \left\{ \frac{\sum_{i=1}^{n} \Delta S_{i,i+1}}{d_{c_n}} \right\}^{-0.7} \left( \frac{T_W}{T_b} \right)^{0.1}
\]

\[
\phi_{\text{Ent.}} = \left( \frac{T_W}{T_b} \right) \left[ 1.59 / \sum_{i=1}^{n} \Delta S_{i,i+1} / d_{c_n} \right]
\]
Curvature Effect

\[ \phi_{Cur.} = \frac{\text{Re}_{CX_{Avg.}}}{\frac{r_{C_n}}{R_{Cur.,n}}} \left( \frac{r_{C_n}}{R_{Cur.,n}} \right)^2 \pm 1/20 \]

where \( r_{C_n} \) is the hydraulic radius of cooling channel, \( R_{Cur.,n} \) is the radius of curvature, the sign (+) denotes the concave curvature and the sign (-) denotes the convex one.
Swilers for Enhancing Heat Transfer

\[ \frac{Gr}{Re^2} = \frac{2d_c \beta_T |T_{cw} - T_{cs}| \tan \alpha}{d_i} \]

\[ \phi_{\text{swiler}} = F \left( 1 + 0.25 \sqrt{\frac{Gr}{Re}} \right) \]

\[ F = 1 + 0.004872 \frac{\tan^2 \alpha}{d_i (1 + \tan^2 \alpha)} \]
Surface Roughness

\[ \frac{Nu}{Nu_{smooth}} = \left( \frac{f}{f_{smooth}} \right)^n \]

\[ n = 0.68 \Pr^{0.215} \]

\[ \frac{1}{\sqrt{f}} = -2.0 \log \left[ \frac{e}{3.7065D} - \frac{5.0452}{\text{Re}_{CX_{Avg.}}} \log \left( \frac{1}{2.8257} \left( \frac{e}{D} \right)^{1.1098} + \frac{5.8506}{\text{Re}_{CX_{Avg.}}^{0.8981}} \right) \right] \]
Pressure Drop

\[ P_{CS_n} = P_{CS_{n-1}} - \left[ (\Delta P_{CS_{n-1,n}})_{f} + (\Delta P_{CS_{n-1,n}})_{M} \right] \]

\[ (\Delta P_{CS_{n-1,n}})_{f} = \frac{f_n}{8 g_c} \left( \frac{\rho_{CS_n} + \rho_{CS_{n-1}}}{d_{c_n} + d_{c_{n-1}}} \right) \left( V_{CS_n} + V_{CS_{n-1}} \right)^2 \Delta S_{n-1,n} \]

\[ (\Delta P_{CS_{n-1,n}})_{M} = \left( \frac{2}{(A_C N)_{n-1} + (A_C N)_n} \right) \frac{W_c^2}{g_c} \left( \frac{1}{(\rho_{CS A_C N})_n} - \frac{1}{(\rho_{CS A_C N})_{n-1}} \right) \]
Typical RTE Results

• Space Shuttle Main Engine (SSME)
• Low-pressure chamber
• High pressure chamber with 200 cooling channels
• High pressure chamber with 150 cooling channels
# Results for SSME

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber pressure</td>
<td>3027 psia</td>
</tr>
<tr>
<td>O/F</td>
<td>6.05</td>
</tr>
<tr>
<td>Contraction ratio</td>
<td>2.66</td>
</tr>
<tr>
<td>Expansion ratio</td>
<td>4.08</td>
</tr>
<tr>
<td>Throat diameter</td>
<td>10.88 inches</td>
</tr>
<tr>
<td>Propellant</td>
<td>GH2-LO2</td>
</tr>
<tr>
<td>Coolant</td>
<td>LH2</td>
</tr>
<tr>
<td>Coolant inlet temperature</td>
<td>95.03R</td>
</tr>
<tr>
<td>Coolant inlet stagnation pressure</td>
<td>6084 psia</td>
</tr>
<tr>
<td>Total coolant flow rate</td>
<td>29.06 lb/sec</td>
</tr>
<tr>
<td>Approximate throat heat flux</td>
<td>80 Btu/in²·sec</td>
</tr>
<tr>
<td>Number of cooling channels</td>
<td>430</td>
</tr>
<tr>
<td>Throat region channel aspect ratio</td>
<td>5</td>
</tr>
</tbody>
</table>
Wall Temperature Distribution for SSME
Temperature Profile (X=-1.4 Inches)

\[ T_{\text{max}} = 1460R \]
Coolant Pressure

Graph showing the relationship between Axial Position (Inches) and Coolant Pressure (Psia). The graph includes two lines:

- Blue line labeled "Static Pressure"
- Red line labeled "Stagnation Pressure"

The y-axis represents Coolant Pressure in Psia, ranging from 4500 to 6300. The x-axis represents Axial Position in inches, ranging from -15 to 10.
### Results for Low Pressure Chamber

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber pressure</td>
<td>450 psia</td>
</tr>
<tr>
<td>O/F</td>
<td>5.8</td>
</tr>
<tr>
<td>Contraction ratio</td>
<td>3.07</td>
</tr>
<tr>
<td>Expansion ratio</td>
<td>5.3</td>
</tr>
<tr>
<td>Throat diameter</td>
<td>8.0 inches</td>
</tr>
<tr>
<td>Propellant</td>
<td>GH2-LO2</td>
</tr>
<tr>
<td>Coolant</td>
<td>LH2</td>
</tr>
<tr>
<td>Coolant inlet temperature</td>
<td>50R</td>
</tr>
<tr>
<td>Coolant inlet stagnation pressure</td>
<td>700 psia</td>
</tr>
<tr>
<td>Total coolant flow rate</td>
<td>15 lb/sec</td>
</tr>
<tr>
<td>Approximate throat heat flux</td>
<td>19 Btu/in²-sec</td>
</tr>
<tr>
<td>Number of cooling channels</td>
<td>240</td>
</tr>
<tr>
<td>Throat region channel aspect ratio</td>
<td>5</td>
</tr>
<tr>
<td>Channel width step changes at X=3.039 inches</td>
<td></td>
</tr>
<tr>
<td>Channel width step changes at X=-4.158 inches</td>
<td></td>
</tr>
</tbody>
</table>
Low Pressure Chamber (unblocked)

\[ T_{\text{max}} = 723R \]

\[ T_c = 91R \]

\[ m_c = 0.0625 \text{ lb/sec} \]

\[ X = -0.618 \text{ inch} \]
Temperature Distribution

![Graph showing temperature distribution with two lines: one for 'One blocked channel' and another for 'No blocked channel'. The x-axis represents Axial Position (inches) and the y-axis represents Wall Temperature (R).]
Temperature Profile

\[ T_{\text{max}} = 1188R \]

\[ X = -0.618 \text{ inch} \]

\[ \dot{m}_c = 0.036 \text{ lb/sec} \]

42% reduction in coolant flow rate

\[ T_c = 207R \]

\[ T_{\text{max}} = 1188R \]
Temperature Profile

$T_{\text{max}} = 1205 \text{R}$

$X = -17.781 \text{ Inch}$

$m_c = 0.036 \text{ lb/sec}$

42% reduction in coolant flow rate
Results for High Pressure Chamber

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber pressure</td>
<td>2000 psia</td>
</tr>
<tr>
<td>O/F</td>
<td>5.8</td>
</tr>
<tr>
<td>Contraction ratio</td>
<td>3.41</td>
</tr>
<tr>
<td>Expansion ratio</td>
<td>6.63</td>
</tr>
<tr>
<td>Throat diameter</td>
<td>2.6 inches</td>
</tr>
<tr>
<td>Propellant</td>
<td>GH2-LO2</td>
</tr>
<tr>
<td>Coolant</td>
<td>LH2</td>
</tr>
<tr>
<td>Total coolant flow rate</td>
<td>6.45 lb/sec</td>
</tr>
<tr>
<td>Coolant inlet temperature</td>
<td>50 R</td>
</tr>
<tr>
<td>Coolant inlet stagnation pressure</td>
<td>3200 psia</td>
</tr>
<tr>
<td>Approximate throat heat flux</td>
<td>77 Btu/in²·sec</td>
</tr>
<tr>
<td>Number of cooling channels</td>
<td>200</td>
</tr>
<tr>
<td>Throat region channel aspect ratio</td>
<td>5-7.8</td>
</tr>
<tr>
<td>Channel width step changes at X=0.947 inches</td>
<td></td>
</tr>
<tr>
<td>X=-3.906 inches</td>
<td></td>
</tr>
</tbody>
</table>
High Pressure Chamber (unblocked)

High pressure chamber
200 cooling channels

\[ T_{\text{max}} = 1058 \text{R} \]

\[ T_c = 122 \text{R} \]

\[ m_c = 0.032 \text{ lb/sec} \]

\[ T_c = 122 \text{R} \]

\[ m_c = 0.032 \text{ lb/sec} \]

\[ T_{\text{max}} = 1058 \text{R} \]
Temperature Distribution
High Pressure, 200 Channels

Wall Temperature (R)

Axial Position (inches)
Temperature Profile

High pressure chamber
200 cooling channels

$T_{\text{max}} = 1479R$

$T_c = 206R$

$X = -0.1 \text{ inch}$

$m_c = 0.024 \text{ lb/sec}$

25% reduction in coolant flow rate
Temperature Profile

High pressure chamber
200 cooling channels

Closed

Open

$T_{\text{max}} = 1580 \text{R}$

$T_c = 645 \text{R}$

$X = -9.38 \text{ inch}$

$\dot{m}_c = 0.024 \text{ lb/sec}$

25% reduction in coolant flow rate
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<td>Coolant inlet temperature</td>
<td>50 R</td>
</tr>
<tr>
<td>Coolant inlet stagnation pressure</td>
<td>2900 psia</td>
</tr>
<tr>
<td>Approximate throat heat flux</td>
<td>75 Btu/in^{2}-sec</td>
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<tr>
<td>Number of cooling channels</td>
<td>150</td>
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<tr>
<td>Throat region channel aspect ratio</td>
<td>5-7.8</td>
</tr>
<tr>
<td>Channel width step changes at</td>
<td>X=0.947 inches</td>
</tr>
<tr>
<td></td>
<td>X=-3.906 inches</td>
</tr>
</tbody>
</table>
High Pressure Chamber (unblocked)

High pressure chamber
150 cooling channels

$T_{\text{c}} = 119R$

$\dot{m}_c = 0.043 \text{ lb/sec}$

$T_{\text{max}} = 1211R$

$X = -0.1 \text{ inch}$
Temperature Distribution
High Pressure, 200 Channels

Wall Temperature (R)

Axial Position (inches)
High pressure chamber
150 cooling channels

$T_{\text{max}} = 1766 \text{R}$

$T_c = 206 \text{R}$

$\dot{m}_c = 0.031 \text{ lb/sec}$

28% reduction in coolant flow rate

$X = -0.1 \text{ inch}$
Temperature Profile

High pressure chamber
150 cooling channels

\[ T_{\text{max}} = 1738 \text{R} \]
\[ T_c = 651 \text{R} \]
\[ \dot{m}_c = 0.031 \text{ lb/sec} \]

28% reduction in coolant flow rate

X = -9.38 inch
Future Work for Expanding Capabilities of RTE

- Modeling hypersonic air breathing engines.
- Incorporating other cooling channel shapes.
- Developing a CFD model for coolant flow analysis.
- Converting the code to a design tool.
Hypersonic Engines (Scramjets)

- LH2 Tank
- Hydrogen/Air Heat Exchanger
- Ram Air
- Film Cooling
A Typical Cooling Panel for Scramjet

Coolant Flow (e.g., LH2)
Combustor and Nozzle Flow Passage for Hypersonic Engines

Mostly rectangular cross-section

Some axisymmetric passages
Use RTE to evaluate cooling channel wall heat flux distribution using coolant heat transfer and pressure drop correlations.

Input cooling channel wall heat flux distribution into the CFD model and calculate cooling channel wall (surface) temperature distribution.

Use RTE and evaluate a new cooling channel wall heat flux distribution using cooling channel wall temperatures from the CFD model.

Are cooling channel wall temperatures of two consecutive iterations close?

Yes: Stop, results converged.

No: Repeat the process.
Different Cooling Channel Shapes

- **Tubular Channels**
  - Bonding material

- **Truncated Oval Channels**
  - Close-up
  - Coating
  - Coolant injection nozzles

- **Increased hot-gas side surface area**
- **Cooling channel with transpiration injection**
Make two initial guesses for inlet pressures \((P_{i1} \text{ and } P_{i2})\) and determine the corresponding exit pressures \((P_{e1} \text{ and } P_{e2})\).

Evaluate a revised inlet pressure using the following equation:

\[
P_{i3} = P_{i1} + (P_e - P_{e1}) \frac{P_{i2} - P_{i1}}{P_{e2} - P_{e1}}
\]

Then use the code to calculate the (exit pressure for).

\[\frac{|P_e - P_{e3}|}{P_e}\]

very small?

If \(h_1 < h_2\), then \(P_{i2} = P_{i3}\) and \(P_{i1}\) remains unchanged.

If \(h_2 < h_1\), then \(P_{i1} = P_{i3}\) and \(P_{i2}\) remains unchanged.

Stop, the results converged, \(P_{i3}\) is the inlet pressure.
Design for Aspect Ratio

Breaks the cooling channel width interval into a number of increments (i.e. \(w_1, w_2, w_3, \ldots, w_n\), where \(w_1\) is the minimum width and \(w_n\) is the maximum width).

For each width value a procedure similar to that shown before will be used to determine the corresponding cooling channel height that yields the desired surface temperature at the throat. The resulting output will be \(n\) possible solutions, \((w_1, h_1), (w_2, h_2), \ldots (w_n, h_n)\), from which the most feasible design from manufacturing point can be selected.
To obtain a copy of RTE contact Dr. Mohammad Naraghi

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